Linear Algebraic Modeling of Power Flow in the HMPT500-3 Transmission

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ABSTRACT

The HMPT500-3 is a split torque path hydrostatic / mechanical continuously variable transmission used in the Bradley Fighting Vehicle. Power transmission and steering is accomplished through the interaction of six planetary gear sets and two variable displacement hydrostatic pump / motor units (HSUs). Power flow in the HMPT500-3 is extremely complex, with numerous feedback paths within the planetary gear train. Without a clearly defined power flow path from gear set to gear set, the analysis cannot be handled in the conventional stepwise manner. The complete speed and torque equation sets must be solved simultaneously for all components. A linear algebraic approach was developed to model forward operation of the HMPT500-3 without steering. The left and right HSUs are lumped in a single unit, and the steering differential is ignored. A reduced set of 14 simultaneous equations for speed and 14 simultaneous equations for torque enable modeling of the ideal (lossless) power flow in the transmission for all forward ranges. Transmission losses are then estimated based on the speed and torque for each individual component, allowing the transmission overall efficiency to be calculated.

INTRODUCTION

The HMPT500-3 is a split torque path hydrostatic / mechanical continuously variable tracked vehicle transmission. Power transmission and steering is accomplished through the interaction of six planetary gear sets and two variable displacement hydrostatic pump / motor units (HSUs), connected through intermediate shafts and gears. The desired transmission speed ratio is achieved by first selecting a discrete mechanical range with clutches and brakes and then varying the HSU displacement to either add or subtract speed from the outputs in a continuous manner. The left and right HSU displacements are varied identically to control vehicle forward speed

and differentially to provide steering. Except in reverse, only part of the HSU displacement capacity is used for vehicle speed control, with the remainder reserved for steering.   A simplified schematic for the transmission is shown in fig. 1.

The HMPT500-3 is currently used in the Bradley Fighting Vehicle (BFV) family and Multiple Launch Rocket System (MLRS). It is also proposed for use in the Paladin Integrated Management (PIM) self-propelled howitzer and BFV derived Armored Multipurpose Vehicle (AMPV). Weight growth of these vehicles is exceeding the capacity of the current HMPT500-3 transmission, and will require design changes to increase torque capacity and ratio range. The

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desire to increase vehicle fuel economy and reduce the heat-rejection burden to the cooling system also makes efficiency improvements desirable. Automotive performance models of the BFV to date have used a measured efficiency vs. speed-ratio curve, rather than a detailed physics based model of the transmission. While a detailed report on the development of the HMPT500 exists [1], it is for an earlier configuration without the third range planetary, and does not describe the computational methods used. Only brief abstract level reports covering development of the current HMPT500-3 have been found [2]. A new mathematical model of the current transmission needed to be developed to predict the component level performance impact of design changes, and provide input into the vehicle level models.

With the exception of first and reverse ranges, which are purely hydrostatic, power flow in the HMPT500-3 is extremely complex with numerous feedback paths within the planetary gear train. Without a clearly defined power flow path from gear set to gear set, the analysis cannot be handled in the stepwise manner used for conventional transmissions. The complete speed and torque equation sets must be solved simultaneously for all planetary gear sets and HSUs. A linear algebraic approach was developed to model forward operation of the HMPT500-3 without steering. In this model the left and right HSUs are lumped in a single unit, and the steering differential is replaced by a fixed ratio. A reduced set of 14 simultaneous equations for speed and 14 simultaneous equations for torque enable modeling of the ideal (lossless) power flow in the transmission for all forward ranges. Transmission losses can then be estimated based on the speed (e.g. "spin losses") and torque for each individual component. The net transmission output after deducting all losses allows the transmission overall efficiency to be calculated. Expansion of the model to 22 simultaneous speed / torque

equations will allow full transmission operation with steering to be modeled.
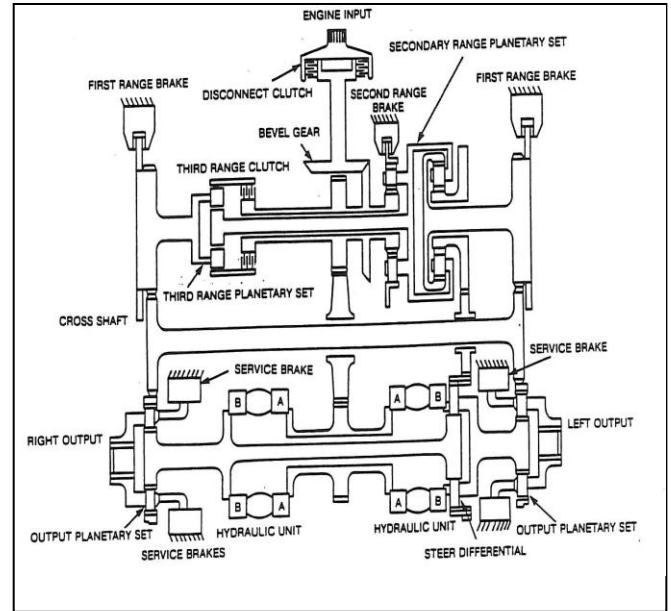
Figure 1, Schematic

Governing Equations of the Model

The simplified model for forward operation of the HMPT500-3 is comprised of four planetary gear sets and one combined HSU. Each planetary set is further divided into a sun gear, a planetary gear carrier, and a ring gear. The HSU is divided into hydraulically coupled "A" and "B" end components, where the "A" end has variable displacement controlled by the stroke of a hydraulic actuator, and the "B" end is fixed displacement. These 14 individual elements require 14 independent equations for both rotational speed and torque to be solved. Intermediate shafts and gears are handled implicitly in the governing equations. In the following equations N refers to the number of gear teeth, S to speed (rpm), T to torque (lb-ft) and P to Power (hp). Gear tooth numbers were obtained from the latest drawings based on part numbers from [3], and should be applicable to the current transmission configuration.

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Each planetary gear set is governed by one speed and two torque equations. The rotating speeds are related by equation (1), with indices 1, 2, 3 referring to the sun, carrier and ring respectively. All equations will be given in homogeneous form as used in the speed and torque matrices.

$$\frac{N_1}{(N_1+N_3)} * S_1 - S_2 + \frac{N_3}{(N_1+N_3)} * S_3 = 0 \quad (1)$$

The planetary torque balance is described by equations (2), (3).

$$\frac{1}{N_1} * T_1 + \frac{2}{(N_1+N_3)} * T_2 + \frac{1}{N_3} * T_3 = 0 \quad (2)$$

$$\frac{1}{N_1} * T_1 - \frac{1}{N_3} * T_3 = 0 \quad (3)$$

The HSU "A" and "B" ends are governed by one speed (4) and one torque (5) equation, with Z defining the "A" end stroke actuation, and neglecting any leakage flow.

$$Z * S_A - S_B = 0; \quad -1 \le Z \le 1 \quad (4)$$

$$T_A + Z * T_B = 0 \quad (5)$$

The sign convention for speeds and torques is positive for power being input to an element, and negative for power being extracted. This results in a net power balance of zero for each planetary set and HSU.

Intermediate gears and shafts connecting planetary and HSU elements are handled by constraint equations. Intermediate gear ratios are implicit in the constraints. For each pair of elements m, n with an imposed speed ratio "R" not defined by the planetary or HSU equations, there is an equation of the form (6).

$$S_m - R_{m,n} * S_n = 0 \quad (6)$$

For each intermediate gear or cross shaft the sum of applied torques from all planetary and HSU elements is zero (7).

$$\sum \frac{T_m}{R_{m,n}} = 0 \quad (7)$$

Elements that have clutches or brakes are treated differently in the constraints for each forward range. A braked element has a zero speed value imposed, but still produces a reaction torque. An element that is "floating" through an open clutch or brake has a zero torque value imposed, but is free to rotate. The only non-homogeneous constraints are for the imposed input speed and torque from the engine. The resulting speed and torque matrix equations for each forward range and HSU stroke are solved separately (8), (9), and the resulting ideal (lossless) power flow calculated (10).

$$[a] * \{S\} = \{b\} \quad (8)$$

$$[c] * \{T\} = \{d\} \quad (9)$$

$$P_i = \frac{T_i * S_i}{5252} \quad (10)$$

Losses are deducted for the input driven makeup / auxiliary pumps based on the flow-rates and pressures in [4] to calculate the net engine torque into the transmission. All other losses are calculated after the ideal speed and torque matrices have been solved. Simple mechanical throughput losses are estimated by multiplying the power solution vector with a diagonal loss factor matrix (11).

$$\{P_{loss}\} = \begin{bmatrix} f_1 & & \\ & f_2 & \\ & & f_n \end{bmatrix} * \{P_{ideal}\} \quad (11)$$

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Additional no-load spin losses are applied for the HSUs based on the "A" end speed and stroke Z as provided in [5]. The net transmission output after deducting all losses allows the transmission overall efficiency to be calculated.

Model Output

All results presented are for input conditions of 2600 rpm, 1010 lb-ft input torque (500HP net input after fan PTO). Detailed output is shown in figures 3 – 5 at the end of the paper for legibility. Model output consists of the speed, torque and power of each planetary and HSU element over the HSU stroke range shown in columns 1 - 3. Plots are grouped by row for 2nd range, 3rd range and Output planetary gears, followed by the HSU (refer to fig. 1 for location).

In first range (fig 3.) engine speed is constant, providing constant input speed to *Second- Range Planetary Sun1* gear and *HSU-A* (*italic* text refers to plot line-series). Engine torque is ramped up from zero to maximum and then held constant as the HSU stroke is advanced from 0 to +0.85 to limit pressure in the HSU. First range brakes are applied, holding *Second-Range Planetary Ring2*, *Third-Range Planetary Carrier* and *Output Planetary Ring* gears. The second range brake and third range clutch are released, so *Second-Range Planetary Ring1* and *Third-Range Planetary Ring* gears transmit no torque. As a result, the remaining ungrounded second and third range planetary elements rotate, but do not transmit torque. As the maximum HSU stroke is reached, *Second-Range Planetary Ring1* has reached ~0 rpm, ready for the second range brake to engage. *Output Planetary Carrier* (output from the transmission to the vehicle final drives) speed increases from zero to ~440 rpm. The rated output speed in first range calculated from [6], corrected for the different stroke limit in reverse, is shown for reference (reverse speed was the only available reference point for the purely hydrostatic first /

reverse ranges). The *Output Planetary Carrier* power shown is negative due to the sign convention for power leaving an element.

In second range (fig 4.) the engine provides constant input speed to the *Second-Range Planetary Sun1* gear and *HSU-A*. HSU stroke is now reversed from +0.85 to -0.85. This continuity of HSU stroke allows for more rapid shifts. The second range brake is applied, holding *Second-Range Planetary Ring1*. This makes the first stage a fixed ratio to engine speed, so *Second Range Carrier-1* and *Carrier-2* (which are joined) rotate at constant speed. The first range brakes and third range clutch are released, so *Third-Range Planetary Ring* gear transmits no torque. As a result the other third range planetary elements freewheel without transmitting any torque. As the minimum HSU stroke is reached, *Third-Range Planetary Ring* gear reaches synchronous speed with the input cross shaft, ready for the third range clutch to engage. *Output Planetary Carrier* speed increases to ~1570 rpm.

In third range (fig 5.) the third range clutch is engaged, so *Second-Range Planetary Sun1*, *Third-Range Planetary Ring* and *HSU-A* are all driven at constant speed by the engine. The first and second range brakes are released. As a result *Second-Range Planetary Ring1* gear transmits no torque, and the first stage of the second range planetary set freewheels. The second stage is active with feedback to *Second Range Planetary Sun2,* so both second and third range planetary sets do transmit power in this range. *Output Planetary Carrier* speed increases to ~3300 rpm as the HSUs are stroked forwards again from -0.85 to +0.85. The rated output speed in high range calculated from [6] is shown for reference.

The detailed results show that the model correctly duplicates the HMPT500-3 kinematics using a consistent set of equations, with only minor matrix row variations to apply or release the

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appropriate clutches / brakes. Output speed increases linearly from zero to maximum rated speed as the transmission shifts through the ranges with constant input speed. Some of the key features of the HMPT series are demonstrated, including HSU stroke continuity and planetary speed synchronization during shifts from range to range [1]. In the idealized model results, power is conserved not only in each planetary set and HSU, but in the entire transmission. The transmitted power levels of certain components exhibit the interesting phenomenon of "reactive power", in which circulating power levels can exceed the input power [7]. This has a negative impact on mechanical efficiency.

To calculate overall efficiency, estimated power transmission loss factors were applied of 0.5% for each gear mesh and 5% for each HSU pump / motor element. Additional makeup / auxiliary pump and HSU no-load spin-loss powers were also deducted. The resulting net transmission output produces the efficiency curve shown (fig 2a.) for a power input of 500hp. The calculated efficiency curve shows very good agreement with the measured curve in second and third range, but only fair agreement in first range. The shape of the efficiency curve can be directly correlated to both the hydrostatic power ratio in the transmission, and the mechanical reactive power ratio (fig 2b.). The transmission efficiency is lowest where these values are high, particularly at the beginning of second range operation.

CONCLUSIONS & RECOMMENDATIONS

The model correctly duplicates the HMPT500-3 kinematics in forward operation using a consistent set of linear equations, with only minor variations to apply or release the appropriate clutches / brakes for each transmission range. Output RPM is correctly predicted for each range as HSU stroke is varied, and synchronous shifting is achieved from range to range. In the idealized

model results, power is conserved not only in each planetary set and HSU, but in the entire transmission. The use of throughput loss factors and spin-losses allows calculation of the net transmission power output, and produces a reasonable approximation of the measured efficiency curve. Modeling of first range operation may be improved by accounting for non-linear losses such as leakage flow in the HSU. In this range the planetary train plays no significant role, and there is no need to solve the complete linear equation set.
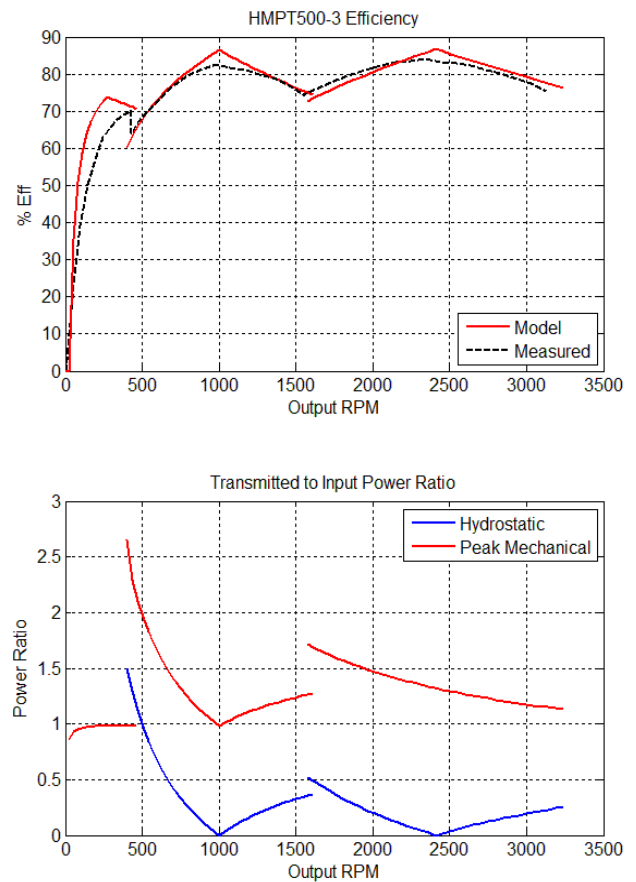

Figure 2a, 2b Efficiency & Power Ratios

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The linear model can be expanded to model complete transmission operation, including steering. The full model of the HMPT500-3 is comprised of six planetary gear sets and two HSUs. The resulting 22 individual elements will require 22 independent equations for both rotational speed and torque to be solved. The complete HMPT500-3 transmission model will be used for power-train models that interface with multi-body vehicle dynamics models and require steering for driver-in-the-loop experiments. The model may also be used for preliminary studies of potential transmission improvements to increase ratio range and efficiency, such as changes to the input bevel ratio and use of alternative speed-ratio generators in place of the HSUs.
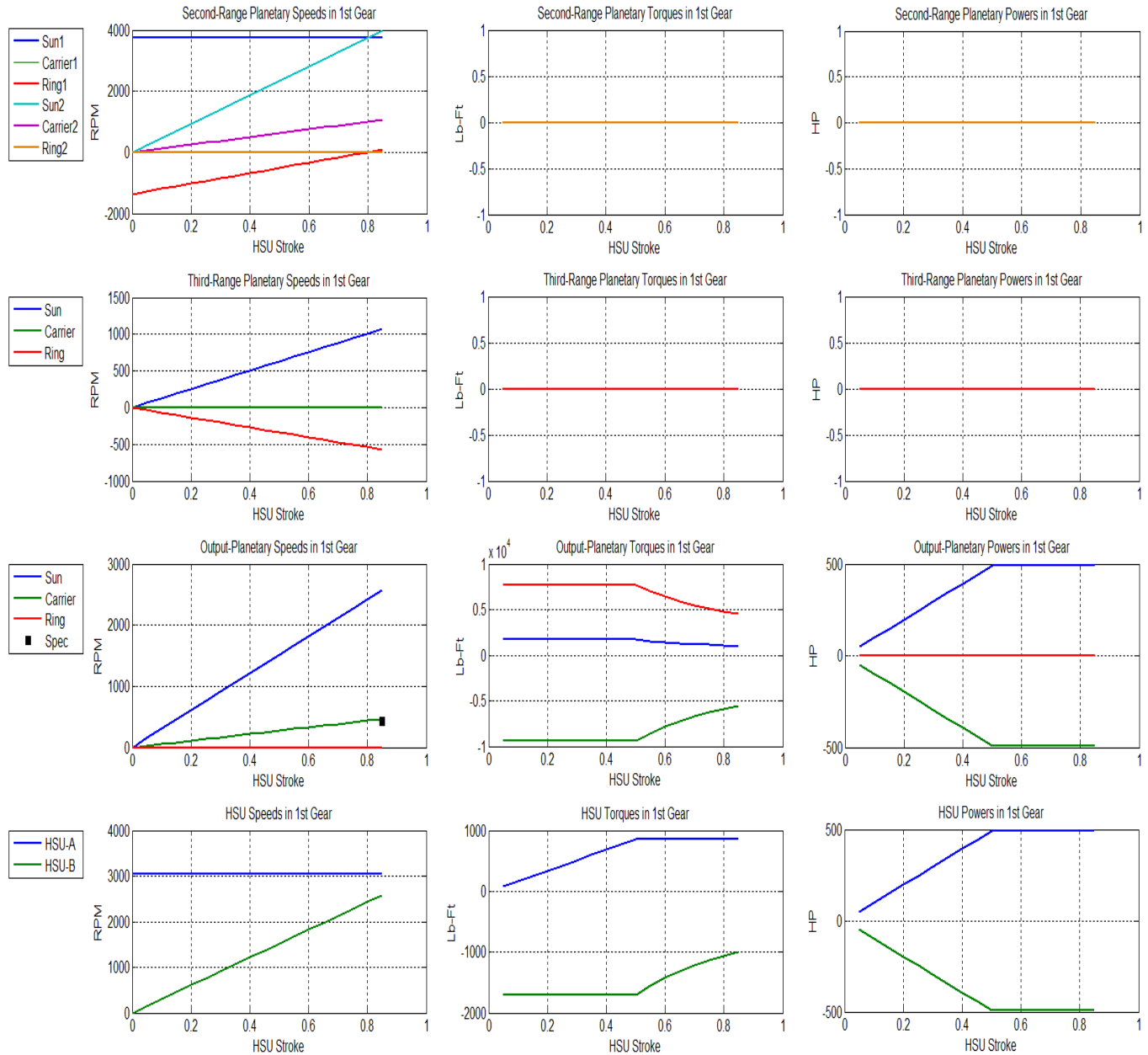
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REFERENCES

[1]R. Northup, "Development and Test of HMPT-500", Final Engineering Report, Contract DAAE07-72-C-0200, General Electric Ordnance Systems, Dec 1974.

[2]J. Lenahan, "Extended 3-Range Transmission", IR&D Summary, General Electric Co., Jun 1982

[3]National Maintenance Work Requirement 9-2520-281 with Repair Parts & Special Tools List, "…Transmission, Crossdrive, Hydro-mechanical, HMPT500 Series…", TACOM LCMC, Oct 2009

[4]Quality Assurance Provision (QAP) 12380360, Makeup / Auxiliary Pump

[5]QAP 12446296, Hydraulic Assembly, LH

[6]Performance Specification 12446502, Rev G, "Transmission, Cross-Drive, Hydromechanical HMPT500-3ECB", Sep 1998; 3.3.9.6 '*Maximum Forward Ratio*', 3.3.9.7 '*Maximum Reverse Ratio*'

[7]G. Lechner, H. Naunheimer, "Automotive Transmissions: Fundamentals, Selection, Design and Application", Springer, Berlin, 1999

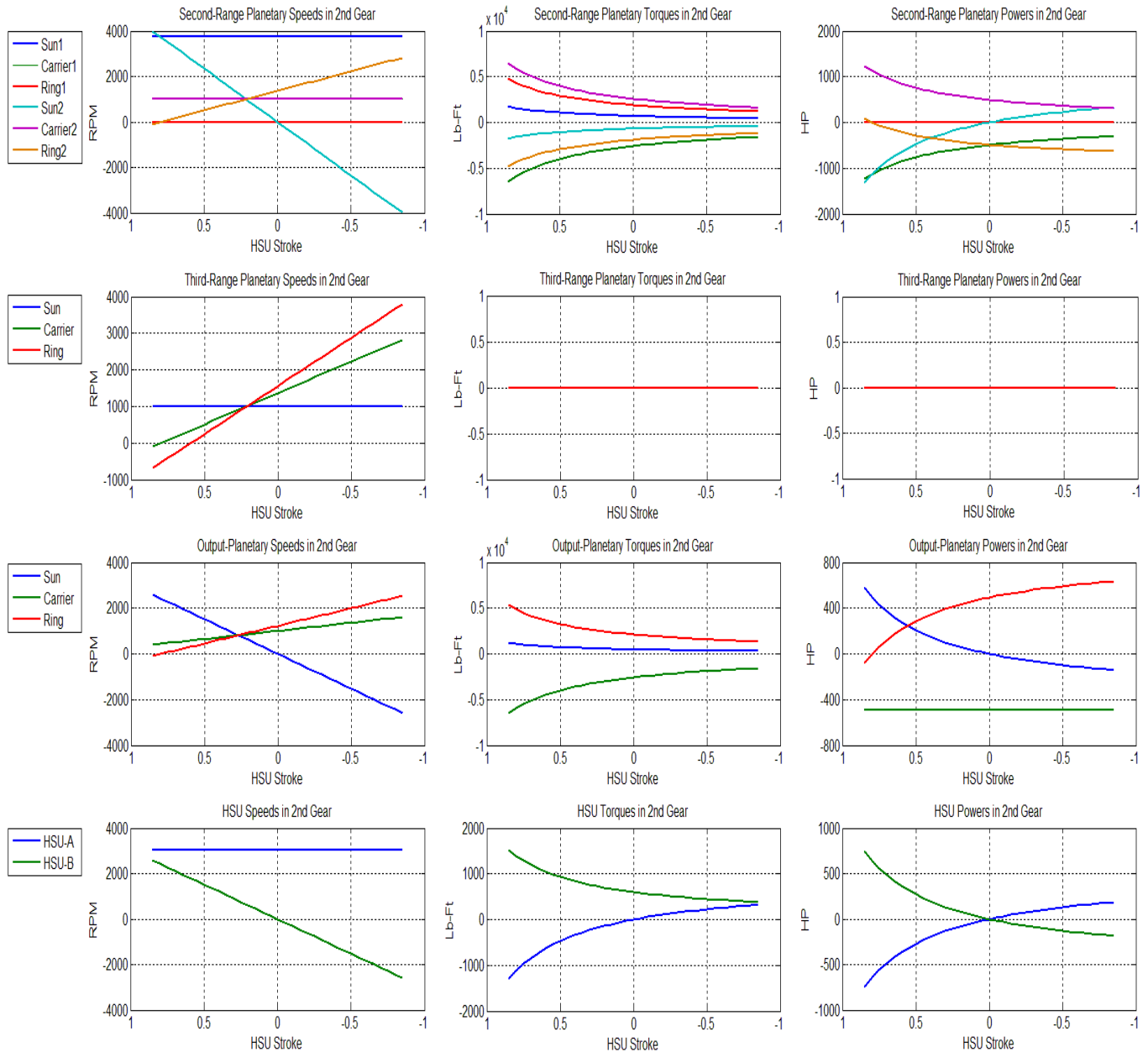**Linear Algebraic Modeling of Power Flow in the HMPT500-3 Transmission**
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Figure 3, First Range Output

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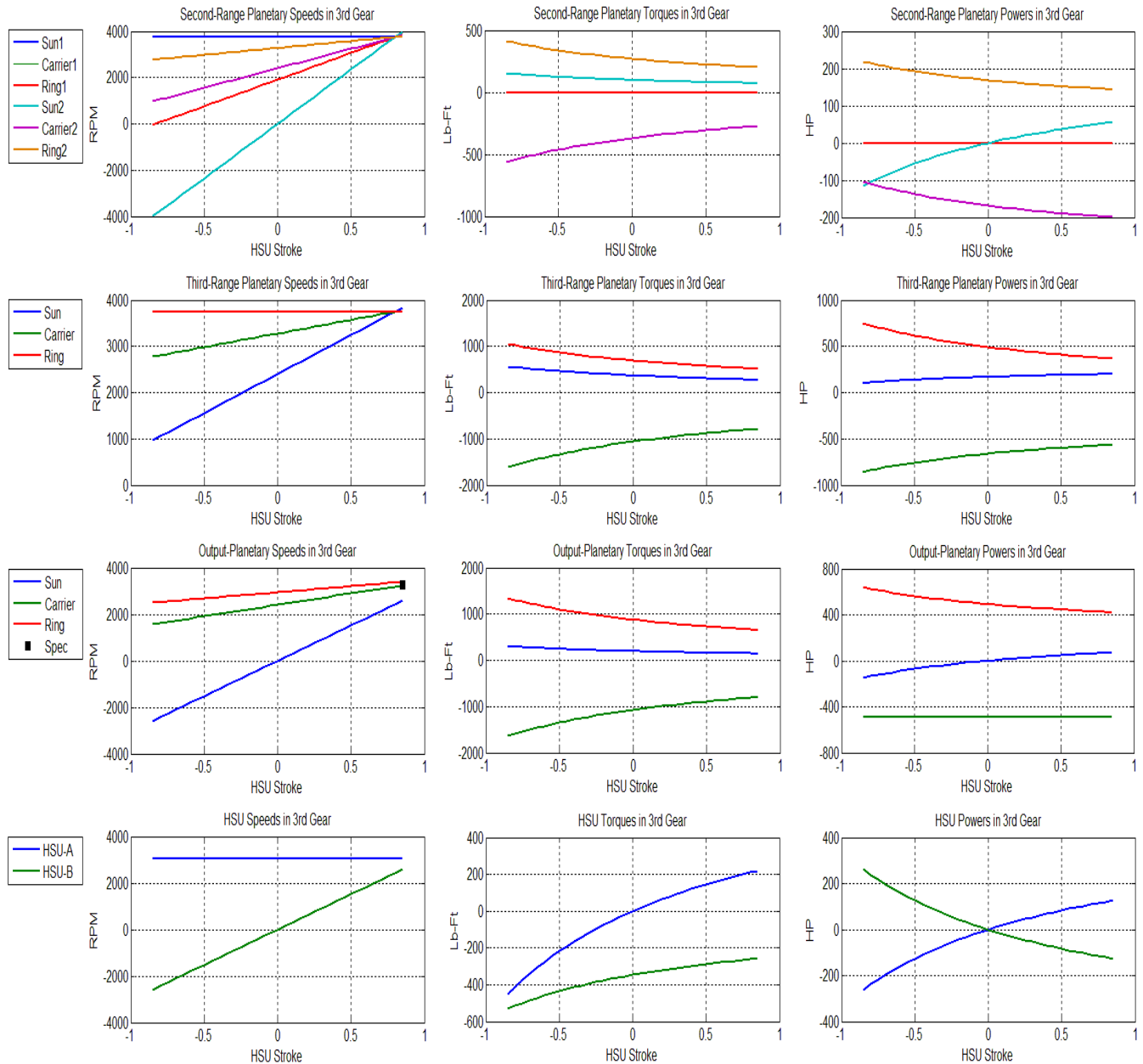**Figure 4, Second Range Output**

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Figure 5, Third Range Output

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